

Analysis 1, Summer 2023

List 0

Algebra review

1. Which of the following are true for all real values of the variables (or, at least, all real values for which both sides of the equation are defined)?

- | | |
|---|--|
| (a) $2x = x + x$ | (h) $k^{-2} = \frac{1}{k^2}$ |
| (b) $2(x + y) = 2x + y$ | (i) $x^{a+2} = x^a \times x^2$ |
| (c) $(x - y)^2 = x^2 - 2xy + y^2$ | (j) $\sqrt{a + b} = \sqrt{a} + \sqrt{b}$ |
| (d) $\frac{6 + a}{2} = 3 + \frac{a}{2}$ | (k) $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ |
| (e) $-(y + 2) = -y + 2$ | (l) $\log(a + b) = \log(a) + \log(b)$ |
| (f) $-(a + b)^2 = (-a + b)^2$ | (m) $\log(a \cdot b) = \log(a) + \log(b)$ |
| (g) $x^3 + 3x = x + x$ | (n) $\log(a \cdot b) = \log(a) \cdot \log(b)$ |

2. Compute the following values:

- | | | |
|----------------------|-----------------------|---------------------|
| (a) $\cos(30^\circ)$ | (g) $\sqrt{10000}$ | (m) $\log_9(3)$ |
| (b) $\cos(45^\circ)$ | (h) $10000^{1/2}$ | (n) $\log_k(1)$ |
| (c) $\cos(60^\circ)$ | (i) $\sqrt[4]{10000}$ | (o) $\ln(1)$ |
| (d) $\cos(\pi/3)$ | (j) $625^{-1/2}$ | (p) $\ln(\sqrt{e})$ |
| (e) $\cos(\pi/2)$ | (k) $\log_7(49)$ | (q) $\log_4(16^3)$ |
| (f) $\sin(8\pi/3)$ | (l) $\log_4(1024)$ | |

3. Re-write $\frac{x^4}{\sqrt{x^{13}}}$ in the form x^\square . (That box is not a mistake in the file.)

4. Re-write each of the following in the form $\square x^\square$, or state that this is not possible. You can assume $x > 0$ if necessary.

- | | | |
|-----------------------|-----------------------------|--|
| (a) $x^3 \cdot x^7$ | (g) $(x^3)^7$ | (n) $\ln(e^x)$ |
| (b) $x^3 x^7$ | (h) $3x^5 + (x^5)^2$ | (o) $\ln(e^{6x})$ |
| (c) $x^3 + x^7$ | (i) $3x^{10} + (x^5)^2$ | (p) $e^{9\ln(x)}$ |
| (d) $x^3 - x^7$ | (j) $\sqrt{x^4}$ | (q) $e^{9\ln(x)+1}$ |
| (e) x^3/x^7 | (k) $\sqrt{x^6 + x^4}$ | (r) $e^{9\ln(x)} + 1$ |
| (f) $\frac{x^3}{x^7}$ | (l) $\sqrt{x^6} + 8x^3$ | (s) $\frac{\sqrt{4e^{\ln(9x)+10}}}{e^5}$ |
| | (m) $x^3 \cdot \sqrt[3]{x}$ | |

5. Re-write $\log_{10}(2^9)$ using the natural logarithm (\ln).

6. Expand each of the following:

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| (a) $(a - b)^2$ | (b) $(1 + x)^2$ | (c) $(2 - x)^3$ | (d) $(6 + h)^3$ |
|-----------------|-----------------|-----------------|-----------------|

7. Simplify $\frac{(6 + h)^3 - 216}{h}$ if $h \neq 0$.

8. If a point on the line

$$y = -\frac{1}{3}(x - 6) + 8$$

has an x -value of 15, what is its y -value?

9. If a point on the line

$$y = -\frac{1}{3}(x - 6) + 8$$

has an x -value of 6, what is its y -value?

10. Graph each of the following:

(a) $y = 3(x - 1) + 2$

(c) $y - 2 = 3(x - 1)$

(e) $3x - y = 1$

(b) $y = 3x - 1$

(d) $y + 1 = 3x$

(f) $x = (y + 1)/3$

11. Give an example of a point that is on the line

$$y - 17 = 38(x - 12).$$

12. Describe the shape of $y = 7$ in words. Describe $x = -2$ in words.

13. Give an equation for the line through the point $(-6, 5)$ with slope 2.

14. Give an equation for each of the following:

(a) the line through $(1, 3)$ with slope 5.

(b) the line through $(0, -9)$ with slope $\frac{2}{5}$.

(c) the line through $(-4.2, 6.1)$ with slope 8.88.

(d) the line through $(5, 1)$ with slope -3 .

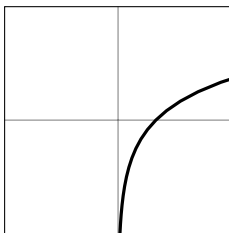
15. A line passes through both $(4, 4)$ and $(8, 2)$. What is its slope?

16. Give an equation for the line through $(1, 7)$ and $(10, -6)$.

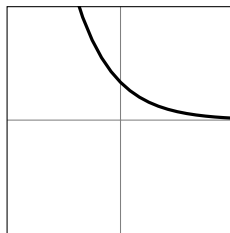
17. For $f(x) = \begin{cases} x + 1 & \text{if } x < 4 \\ x^2 & \text{if } x \geq 4 \end{cases}$, what is the value of $f(4)$?

18. Draw a graph of the piecewise function $\begin{cases} x & \text{if } x < 1 \\ 2x & \text{if } 1 \leq x \leq 2 \\ 6 - x & \text{if } x > 2 \end{cases}$.

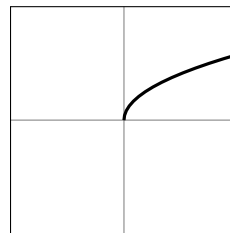
19. Match the functions with the graph: (a) e^x (b) \sqrt{x} (c) $\ln(x)$ (d) e^{-x}



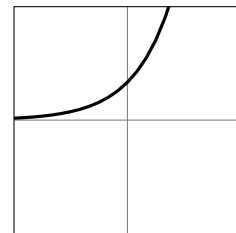
(I)



(II)



(III)



(IV)